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**INFLUENCE OF ACOUSTIC PRESSURE WAVE ON FLAT GYRO  
SUSPENSION ELEMENTS**

**ВПЛИВ АКУСТИЧНОЇ ХВИЛІ ТИСКУ НА ПЛОСКІ ЕЛЕМЕНТИ  
ПІДВІСУ ГІРОСКОПУ**

**Summary.** *The nature of the interaction of sound with an obstacle is revealed. The dynamic characteristics of the plate under an acoustic pressure wave are investigated. A thin isotropic plate was chosen as a mechanical model for the interaction of sound with a flat barrier. The questions of the influence of mechanical impedance for the symmetric and antisymmetric components of sound pressure are considered. Numerical analysis of bending vibrations of the shadow side of the plate is carried out. The value of the lengths of the modulating waves of the main bending vibrations is established*

**Key words:** *acoustic pressure wave, flat elements of the gyroscope suspension, a thin isotropic plate.*

**Анотація.** Розкривається природа взаємодії звуку із перешкодою. Досліджуються динамічні характеристики пластини за акустичної хвилі тиску. Як механічна модель взаємодії звуку з плоскою перешкодою обрана тонка ізотропна пластина. Розглянуто питання впливу механічного імпедансу для симетричної та антисиметричної складових звукового тиску. Проведено чисельний аналіз згинальних коливань тіньової сторони пластини. Встановлено значення довжин модулюючих хвиль основних згинальних коливань.

**Ключові слова:** акустична хвиля тиску, пласкі елементи підвісу гіроскопа, тонка ізотропна пластина.

**Introduction.** Starting to study the nature of the elastic interaction of penetrating acoustic radiation with a flat barrier, first of all, attention should be focused on the choice of a mechanical model of the phenomenon. Studies by a number of authors prove that, in many cases important for practice, in the frequency range below the boundary, the conditions for fixing the plate do not have a decisive effect on its dynamics, and, without distorting the objectivity of the sound transmission pattern, it is permissible to neglect the boundary conditions along the attachment contour, and the computational model can be represented as a plate unlimited in length [1; 2; 3]. It is preferable to consider the distribution of parameters on the boundary surface and at infinity (for an unbounded region) as well as at the origin of coordinates (for a region of limited parameters) as the main boundary conditions.

Such a simplification is permissible, for example, when the plate is hinged with other structural elements, that is, for the case when the bending energy from the oscillating plate is practically not transferred to the contacting elements. These considerations can also be extended to the case when the cylindrical rigidities of the connecting elements are much higher than the bending rigidity of the plate, and it can be stated with full confidence that the

radiation energy is almost completely absorbed by the oscillating plate due to internal friction in the material. In other words, it is assumed that along the contour the plate is hinged to ribs that are absolutely rigid in bending in the direction normal to the plate and, at the same time, have low bending rigidity in planes tangential to the middle surface. Thus, in what follows, we will deal with thin plates of infinite length with large internal absorption.

As for the boundary conditions on the surface of the plates, they consist in the equality of the normal to the plane velocities of the plate and the medium, as well as the pressure. It is obvious that these conditions follow from the requirement of continuous change in pressure and particle velocities at the boundary of two media and are based on the physical impossibility of a pressure jump in infinitely close layers, on the one hand, and a velocity jump, on the other. The latter implies the exclusion of the possibility of the appearance of a displacement jump and, consequently, a discontinuity at the boundary of two media [4].

Based on the foregoing, we take as initial the low rigidity of the plate in the direction of its normal, the hinged fastening along its contour, as well as the absolute rigidity of the plate in bending in the direction of the normal to the surface. If, in addition, we assume that the thickness is 4-6 times greater than the wavelength, then for the analytical description it is legitimate to use the equations of a thin plate.

When compiling a mathematical model of sound transmission, we proceed from the following assumptions - the linear elements of the plate, perpendicular to its middle surface, remain straight during deformation and are set normally to the curved middle surface; no elongation or shear deformation occurs in the median surface; plate bending deformations remain small, elastic and subject to Hooke's law.

Let us focus on a plane monochromatic pressure wave, i.e. a wave with a flat front, the pressure and velocity of particles of the medium in which do not

have a gradient along the front line. In addition, the speed of the particles of the medium in a plane wave will be considered proportional to the pressure at the same time. In practice, a plane wave is considered as an idealization of a wave emitted by a body of finite size, but located at a sufficiently large distance.

The issues of sound transmission through two plates that are not interconnected were considered by *A. London* [5], and a more general theory of the effect of sound on composite structures is described, for example, in monographs [6, 7].

If the noted works studied the issue of the interaction of sound with an obstacle in terms of determining its soundproofing properties, then the main aspect of the research is the dynamic characteristics of the plate under acoustic exposure.

**Thin isotropic plate.** Let us illustrate the solution of the formulated problem using the mechanical model of the interaction of sound with a flat barrier widely used in acoustics (Fig. 1) [8]. Let us assume that an isotropic elastic plate of constant stiffness and unlimited length separates two acoustic half-spaces with the same characteristics, for example, air.

Let, at some point in time, a plane monochromatic sound pressure wave be incident  $\theta$  on the front surface of the plate at an angle

$$P_1 = P_{10} \exp i \left\{ \omega t - k_0 \left[ (z + \delta) \cos \theta + y \sin \theta \right] \right\}, \quad (1)$$

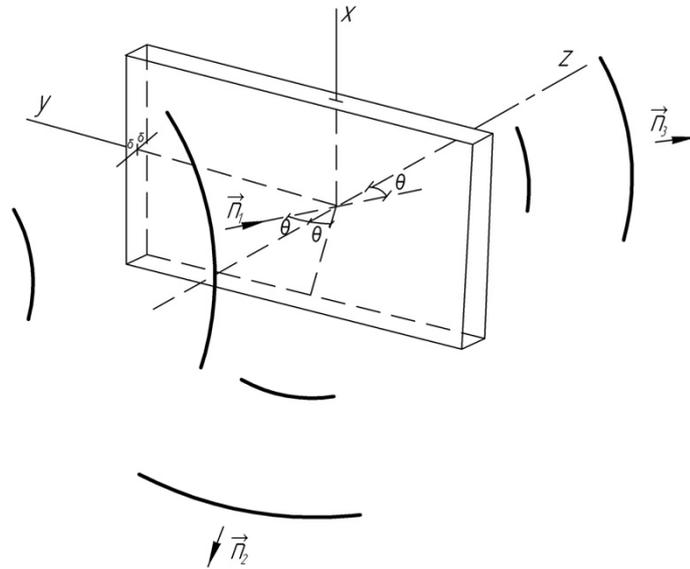
where  $k_0 = \frac{\omega}{c}$  is the wave number;  $\omega$  - circular frequency of oscillations;  $c$  - is the speed of sound in air;  $P_{10}$  - is the pressure amplitude in the sound wave.

The term  $\delta$  at the coordinate  $z$  is introduced for the convenience of further calculations. It does not affect the desired amplitude values of pressure.

For the reflected and transmitted waves we have similarly:

$$\begin{aligned} P_2 &= P_{20} \exp i \left\{ \omega t - k_0 \left[ -(z + \delta) \cos \theta + y \sin \theta \right] \right\}; \\ P_3 &= P_{30} \exp i \left\{ \omega t - k_0 \left[ (z - \delta) \cos \theta + y \sin \theta \right] \right\}. \end{aligned} \quad (2)$$

The movement of the plate occurs only in the plane  $y, z$  and does not depend on the coordinate  $x$ , because along it the pressure on the surface is constant. Thus, there is a flat deformation of the plate.



**Fig. 1. Scheme of the passage of a sound wave through a plane isotropic plate of infinite length**

For this case, the mathematical model of the bending motion in the *Lame* form can be represented by the equations [4] –

$$\begin{aligned} (\lambda + \mu) \frac{\partial \xi}{\partial y} + \mu \nabla^2 V &= \rho_c \frac{\partial^2 V}{\partial t^2}; \\ (\lambda + \mu) \frac{\partial \xi}{\partial z} + \mu \nabla^2 W &= \rho_c \frac{\partial^2 W}{\partial t^2}, \end{aligned} \quad (3)$$

where  $\xi = \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z}$ ;  $\nabla^2 = \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ ;  $V$  и  $W$  – offsets in the direction of the axes  $y$  and  $z$ ;  $\rho_c$  – plate material density (mass per unit volume);  $\lambda$  and  $\mu$  – elastic constants Lamé, which are expressed in terms of Young's modulus  $E$  and Poisson's ratio  $\sigma$  in the following way

$$\lambda = \frac{E\sigma}{(1+\sigma)(1-2\sigma)}; \quad \mu = \frac{E}{2(1+\sigma)}. \quad (4)$$

To solve the system of equations (3), we accept that

$$V = \frac{\partial \varphi}{\partial y} - \frac{\partial \psi}{\partial z}; \quad W = \frac{\partial \varphi}{\partial y} + \frac{\partial \psi}{\partial z}. \quad (5)$$

After substituting relations (5) into equations (3), the latter decompose into two independent equations for the functions  $\varphi$  and  $\psi$  :

$$\nabla^2 \varphi = C_1^{-2} \frac{\partial^2 \varphi}{\partial t^2}; \quad \nabla^2 \psi = C_2^{-2} \frac{\partial^2 \psi}{\partial t^2}, \quad (6)$$

where  $C_1 = [\rho_c^{-1}(\lambda + 2\mu)]^{\frac{1}{2}}$  – P-wave speed;  $C_2 = [\mu\rho_c^{-1}]^{\frac{1}{2}}$  – shear wave speed.

The solution of equations (6) is sought in the form:

$$\varphi(y, z, t) = f_1(z) \exp i(\omega t - k_0 y \sin \theta);$$

$$\psi(y, z, t) = f_2(z) \exp i(\omega t - k_0 y \sin \theta).$$

Substituting the values  $\varphi$  and  $\psi$  into equations (3), after integration, we find their values, then using expressions (5) we determine the functions  $V$  and  $W$  :

$$V = -\left[ (C_1 \exp \alpha_1 z + C_2 \exp(-\alpha_1 z)) i k_0 \sin \theta + \alpha_2 (C_3 \exp \alpha_2 z - C_4 \exp(-\alpha_2 z)) \right] \times \\ \times \exp i(\omega t - k_0 y \sin \theta);$$

$$W = \left[ \alpha_1 (C_1 \exp \alpha_1 z - C_2 \exp(-\alpha_1 z)) - (C_3 \exp \alpha_2 z - C_4 \exp(-\alpha_2 z)) i k_0 \sin \theta \right] \times \\ \times \exp i(\omega t - k_0 y \sin \theta).$$

To establish the values of normal and shear stresses, we use the relations [9]:

$$Z_z = (\lambda + 2\mu) \frac{\partial W}{\partial z} + \lambda \frac{\partial V}{\partial y} =$$

$$= \left[ a_1 (C_1 \exp \alpha_1 z + C_2 \exp(-\alpha_1 z)) - a_2 (C_3 \exp \alpha_2 z - C_4 \exp(-\alpha_2 z)) \right] \times \\ \times \exp i(\omega t - k_0 y \sin \theta);$$

$$Z_y = \mu \left( \frac{\partial V}{\partial z} + \frac{\partial W}{\partial y} \right) =$$

$$= -\left[ a_3 (C_1 \exp \alpha_1 z - C_2 \exp(-\alpha_1 z)) + a_4 (C_3 \exp \alpha_2 z + C_4 \exp(-\alpha_2 z)) \right] \times \\ \times \exp i(\omega t - k_0 y \sin \theta), \quad (7)$$

where:  $a_1 = \alpha_1^2 (\lambda + 2\mu) - \lambda k_0^2 \sin^2 \theta$ ,  $a_2 = 2i\mu\alpha_2 k_0 \sin \theta$ ,  $a_3 = 2i\mu\alpha_1 k_0 \sin \theta$ ,  $a_4 = \mu(\alpha_2^2 + k_0^2 \sin^2 \theta)$ .

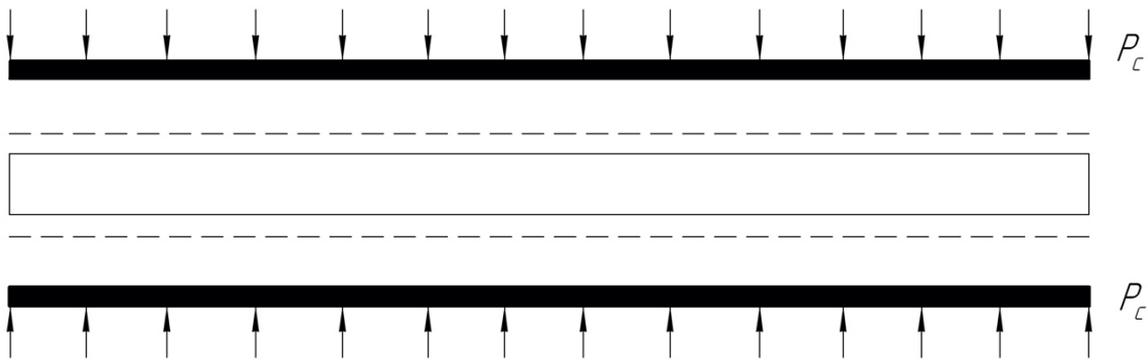
From expressions (1), (2), we determine the sound pressure on the front and shadow sides of the plate:

$$\begin{aligned} (P_1 + P_2)|_{z=-\delta} &= (P_{10} + P_{20}) \exp i(\omega t - k_0 y \sin \theta); \\ P_3|_{z=\delta} &= P_{30} \exp i(\omega t - k_0 y \sin \theta). \end{aligned}$$

Let us represent the values of these pressures as the sum of the symmetric and antisymmetric components (Picture 2, Picture 3) –

$$\begin{aligned} P_c &= \frac{1}{2}(P_{10} + P_{20} + P_{30}) \exp i(\omega t - k_0 y \sin \theta); \\ P_a &= \frac{1}{2}(P_{10} + P_{20} - P_{30}) \exp i(\omega t - k_0 y \sin \theta) \end{aligned}$$

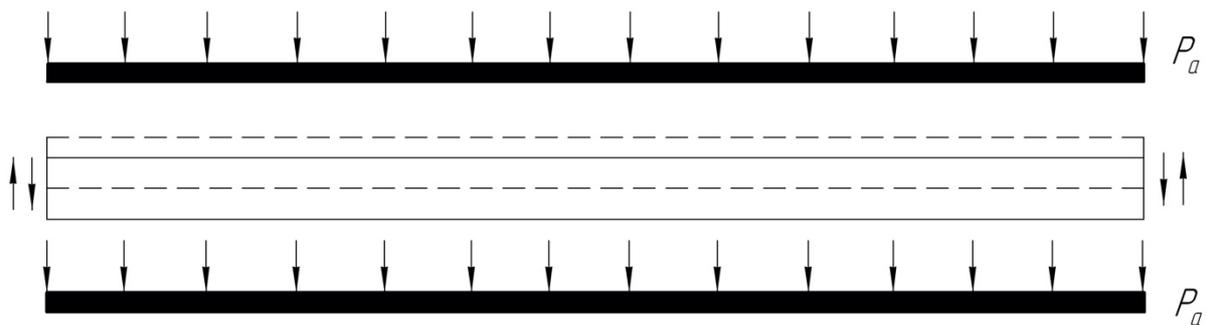
and establish the degree of influence of each of them on the nature of the bending vibrations of the plate.



**Fig. 2. The action of the symmetrical component of sound pressure**

Under the action of the symmetrical component of the sound pressure, the boundary conditions have the form (Picture 4):

$$Z_z|_{z=\pm\delta} = -P_c; \quad Z_y|_{z=\pm\delta} = 0. \quad (8)$$

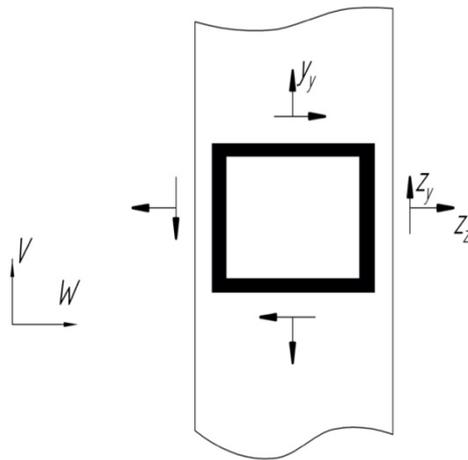


**Fig. 3. The buildup of the plate of the antisymmetric component of the sound pressure**

Using relations (8), we find arbitrary constants of integration  $C_i$  expressions (7). Substituting their values into the formula for lateral displacement  $W$ , we obtain the law of bending vibrations of the plate under the action of the symmetrical component of sound pressure

$$W_c|_{z=\pm\delta} = \mp P_c k_2 \left\{ \omega^{-2} \delta^{-1} \rho_c^{-1} \left[ 4\gamma(\gamma-1)^{\frac{1}{2}} \operatorname{cth} k_2(\gamma-1)^{\frac{1}{2}} - (2\gamma-1)^2 (\gamma-d^2)^{-\frac{1}{2}} \operatorname{cth} k_2(\gamma-d^2)^{\frac{1}{2}} \right]^{-1} \right\}, \quad (9)$$

where  $\gamma = (C_2 c^{-1} \sin \theta)^2$ ;  $k_2 = C_2^{-1} \omega \delta$ ;  $d^2 = (C_2 C_1^{-1})^2 = (1-2\sigma)[2(1-\sigma)]^{-1}$ .



**Fig. 4. Passage of a sound wave through an elastic isotropic layer**

If the antisymmetric component of the excess pressure of the sound frequency acts on the plate, then the boundary conditions take the form:

$$Z_z|_{z=\pm\delta} = -P_a; Z_z|_{z=\delta} = P_a; Z_y|_{z=-\delta} = 0; Z_y|_{z=\delta} = 0,$$

and the displacement value is described by the relation –

$$W_a|_{z=\pm\delta} = k_2 P_a \left\{ \omega^2 \delta \rho_c \left[ 4\gamma(\gamma-1)^{\frac{1}{2}} \operatorname{th} k_2(\gamma-1)^{\frac{1}{2}} - (2\gamma-1)^2 (\gamma-d^2)^{-\frac{1}{2}} \operatorname{th}(\gamma-d^2) \right]^{\frac{1}{2}} \right\}^{-1} \quad (10)$$

When restrictions are met  $\left|k_2(\gamma-1)^{\frac{1}{2}}\right| \leq 0,9$  и  $\left|k_2(\gamma-d^2)^{\frac{1}{2}}\right| \leq 0,9$ ,

implying the preservation of only the first two terms of the expansion into a series of tangents, expressions (9), (10) take the form:

$$W_c|_{z=\pm\delta} = \mp \frac{P_c}{\omega^2 \delta \rho_c} \frac{k_2^2(\gamma-d^2)}{4\gamma(1-d^2)-1} = \mp P_c \delta \frac{1-\sigma^2}{E} \cdot \frac{(C_n c^{-1} \sin \theta)^2 - (1-2\sigma)(1-\sigma)^{-2}}{(C_n c^{-1} \sin \theta)^2 - 1}; \quad (11)$$

$$W_a|_{z=\pm\delta} = \frac{P_a}{\omega^2 \delta \rho_c} \frac{1}{\frac{4}{3}\gamma(\gamma-1)k_2^2\delta^2(1-d^2)-1}. \quad (12)$$

where  $C_n = \left[ E \rho_c^{-1} (1-\sigma^2)^{-1} \right]^{\frac{1}{2}}$  – velocity of longitudinal waves in the plate.

Formula (11) coincides with the law of bending vibrations of a plate at symmetrical pressure, established by Л. И. ЛЯМШЕВЫМ [10].

If  $\gamma \gg 1$ , then the entire first term in the denominator becomes small compared to unity, and formula (12) takes the form:

$$W_a = \frac{P_a}{\omega^2 \delta \rho_c} \frac{1}{\frac{4}{3}(k_2 \delta \gamma)^2(1-d^2)-1} = \frac{2P_a}{\omega^2 m_n} \frac{1}{\frac{D\omega^2}{m_n C^4} \sin^4 \theta - 1}, \quad (13)$$

which is the well-known law of bending vibrations of a thin plate. Here

$D = \frac{2}{3} \frac{E \delta^3}{1-\sigma^2}$  – cylindrical bending stiffness of the plate;  $m_n = 2\delta \rho_c$  – mass per

unit area of the plate. Consequently, the oscillations of the plate on which it falls at an angle  $\theta$  plane sound wave, can be described by the equations of motion of thin plates, if the length of the incident wave trace at  $\gamma \gg 1$  or the length of the transverse wave at  $\gamma \ll 1$  and is less than 3.5...6 layer thicknesses.

Let us further take into account the effect of internal friction in the plate material. In the simplest case, this is achieved by introducing the complex Young's modulus [1; 6; 7; 8], i.e. the hysteresis loop is represented by an ellipse

$E^0 = E(1 + i\eta)$  (here  $E$  – real part of the modulus of elasticity;  $\eta$  – loss factor). By internal friction, we mean the totality of various physical processes in the material that, during deformation, lead to irreversible dissipation of mechanical energy.

In view of the above, the value of the mechanical impedance (the ratio of pressure to the displacement velocity of the plate surface) for the symmetric and antisymmetric components of the sound pressure will be determined by the relations:

$$z_c = \frac{P_c}{\frac{\partial W_c}{\partial t}} = -i\rho_c\omega\delta(z_c^{(1)} + iz_c^{(2)}); \quad z_a = \frac{P_a}{\frac{\partial W_a}{\partial t}} = i\rho_c\omega\delta(z_a^{(1)} + iz_a^{(2)}), \quad (14)$$

where

$$\begin{aligned} z_c^{(1)} &= [4\gamma(1-d^2) - 1]k_2^{-2}(\gamma - d^2)^{-1}; \\ z_c^{(2)} &= \eta_c [4\gamma(1-d^2)(\gamma - 2d^2) + d^2]k_2^{-2}(\gamma - d^2)^{-2}; \\ z_a^{(1)} &= \frac{4}{3}(1-d^2)k_2^2\gamma^2 - 1; \quad z_a^{(2)} = \frac{4}{3}\eta_c(1-d^2)k_2^2\gamma^2. \end{aligned} \quad (15)$$

Comparison of the dynamic and static moduli of elasticity, for example, steel [11], showed that its dynamic rigidity does not differ from the static one. This does not apply to soft materials, where the change in dynamic parameters should be taken into account [2].

Thus, using expressions (11), (12), it is possible to establish the law of motion of any layer of the plate. The inconvenience of the obtained formulas is that the plate displacement is a function of the symmetric and antisymmetric pressure components, and not the pressure amplitude of the incident wave  $P_{10}$ . It is easy to get rid of this shortcoming using the concept of the sound transmission coefficient  $A$  (ratio of pressure amplitudes in the past  $P_{30}$  and falling  $P_{10}$  waves) and sound reflection coefficient  $B = \frac{P_{20}}{P_{20}}$ . According to the calculation model of sound transmission, the total displacement velocity of the plate surfaces under

the action of both symmetric and antisymmetric pressure components will be equal to their sum on the front and their difference on the shadow side of the plates, i.e.

$$\left. \frac{\partial W}{\partial t} \right|_{z=-\delta} = \frac{iP_c}{\omega\delta\rho_c} \frac{1}{z_c^{(1)} + iz_c^{(2)}} + \frac{iP_a}{\omega\delta\rho_c} \frac{1}{z_a^{(1)} + iz_a^{(2)}}; \quad (16)$$

$$\left. \frac{\partial W}{\partial t} \right|_{z=\delta} = \frac{iP_a}{\omega\delta\rho_c} \frac{1}{z_a^{(1)} + iz_a^{(2)}} - \frac{iP_c}{\omega\delta\rho_c} \frac{1}{z_c^{(1)} + iz_c^{(2)}}. \quad (17)$$

From the condition of continuity at the boundary of two media follows the conclusion about the equality of the oscillatory velocity of the plate and the normal component of the velocity of the sound wave. Then the boundary conditions on the surface of the plate can be written as –

$$\left. \frac{\partial W}{\partial t} \right|_{z=-\delta} = \frac{P_1 - P_2}{z_0} \cos\theta; \quad \left. \frac{\partial W}{\partial t} \right|_{z=\delta} = \frac{P_3}{z_0} \cos\theta, \quad (18)$$

where  $z_0 = \rho_0 c$  – specific acoustic resistance of air;  $\rho_0$  – air density;  $V = \frac{P}{z_0}$  – the known relationship between vibrational speed  $V$  and pressure  $P$  for a plane wave in air [3].

Eliminating from equations (16) ... (18) pressure  $P$  and speed  $\frac{\partial W}{\partial t}$ , find the transmission coefficients  $A$  and reflections  $B$  sound –

$$A = \frac{\frac{z_c^{(2)}}{\Delta_c} - \frac{z_a^{(2)}}{\Delta_c} - i \left( \frac{z_a^{(1)}}{\Delta_c} - \frac{z_c^{(1)}}{\Delta_c} \right)}{\left( 1 + \frac{z_c^{(2)}}{\Delta_c} \right) \left( 1 + \frac{z_a^{(2)}}{\Delta_c} \right) - \frac{z_c^{(1)} z_a^{(1)}}{\Delta_c^2} - i \left[ \left( 1 + \frac{z_c^{(2)}}{\Delta_c} \right) \frac{z_a^{(1)}}{\Delta_c} + \left( 1 + \frac{z_a^{(2)}}{\Delta_c} \right) \frac{z_c^{(1)}}{\Delta_c} \right]}; \quad (19)$$

$$B = \frac{1 + \frac{z_a^{(1)} z_c^{(1)}}{\Delta_c^2} - \frac{z_a^{(2)} z_c^{(2)}}{\Delta_c^2} + i \left( \frac{z_a^{(1)} z_c^{(2)}}{\Delta_c^2} + \frac{z_a^{(2)} z_c^{(1)}}{\Delta_c^2} \right)}{\left( 1 + \frac{z_c^{(2)}}{\Delta_c} \right) \left( 1 + \frac{z_a^{(2)}}{\Delta_c} \right) - \frac{z_c^{(1)} z_a^{(1)}}{\Delta_c^2} - i \left[ \left( 1 + \frac{z_c^{(2)}}{\Delta_c} \right) \frac{z_a^{(1)}}{\Delta_c} + \left( 1 + \frac{z_a^{(2)}}{\Delta_c} \right) \frac{z_c^{(1)}}{\Delta_c} \right]}, \quad (20)$$

where  $\Delta_c = z_0 (\omega\delta\rho_c \cos\theta)^{-1}$ .

Taking into account the found values of the coefficients  $A$  and  $B$ , the law of bending vibrations of the plate can be finally written in the form –

$$\begin{aligned}
 W &= \frac{1}{2} \frac{(P_{10} + P_{20} - P_{30})}{\omega^2 m_n} \frac{1}{\frac{D\omega^2}{m_n c^4} \sin^4 \theta - 1} \exp i(\omega t - k_0 y \sin \theta) + \\
 &+ \frac{1}{2} \frac{(P_{10} + P_{20} + P_{30})}{E} \delta(1 - \sigma^2) \frac{\left(\frac{C_n}{c} \sin \theta\right)^2 - \frac{1 - 2\sigma}{(1 - \sigma)^2}}{\left(\frac{C_n}{c} \sin \theta\right)^2 - 1} \exp i(\omega t - k_0 y \sin \theta) = \\
 &= P_{10} \exp i(\omega t - k_0 y \sin \theta) [(1 + B - A)\mu_1 + (1 + B + A)\mu_2] = \\
 &= \frac{P_{10}}{\rho} \exp i(\omega t - k_0 y \sin \theta - \varphi) [\rho_1 \mu_1 \exp i\varphi_1 + \rho_2 \mu_2 \exp i\varphi_2], \quad (21)
 \end{aligned}$$

$$\text{where } \rho = \left\{ \left[ \left( \frac{z_a^{(1)}}{\Delta_c} \right)^2 + \left( 1 + \frac{z_a^{(2)}}{\Delta_c} \right)^2 \right] \left[ \left( \frac{z_c^{(1)}}{\Delta_c} \right)^2 + \left( 1 + \frac{z_c^{(2)}}{\Delta_c} \right)^2 \right] \right\}^{\frac{1}{2}};$$

$$\rho_1 = \left[ \left( 1 + \frac{z_a^{(2)}}{2\Delta_c} \right)^2 + \left( \frac{z_a^{(1)}}{\Delta_c} \right)^2 \right]^{\frac{1}{2}}; \quad \rho_2 = \left[ \left( 1 + \frac{z_c^{(2)}}{2\Delta_c} \right)^2 + \left( \frac{z_c^{(1)}}{\Delta_c} \right)^2 \right]^{\frac{1}{2}};$$

$$\varphi_1 = \operatorname{arctg} \left[ -\frac{z_c^{(1)}}{\Delta_c} \left( 1 + \frac{z_a^{(2)}}{2\Delta_c} \right)^{-1} \right]; \quad \varphi_2 = \operatorname{arctg} \left[ -\frac{z_c^{(1)}}{\Delta_c} \left( 1 + \frac{z_c^{(2)}}{2\Delta_c} \right)^{-1} \right];$$

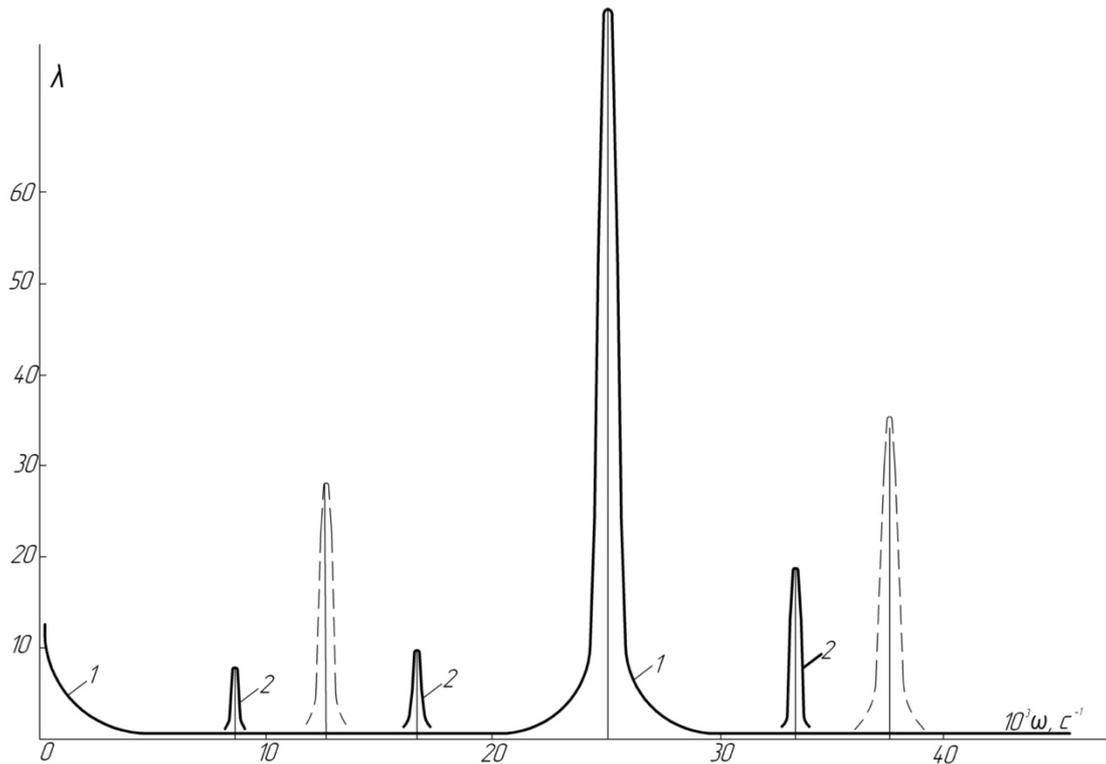
$$\varphi = \operatorname{arctg} \left[ \frac{z_a^{(1)}}{\Delta_c} \left( 1 + \frac{z_c^{(2)}}{\Delta_c} \right) + \frac{z_c^{(1)}}{\Delta_c} \left( 1 + \frac{z_a^{(2)}}{\Delta_c} \right) \right] \left[ \frac{z_a^{(1)} z_c^{(1)}}{\Delta_c^2} - \left( 1 + \frac{z_c^{(2)}}{\Delta_c} \right) \left( 1 + \frac{z_a^{(2)}}{\Delta_c} \right) \right]^{-1};$$

$$\mu_1 = \left[ \omega^2 m_n \left( \frac{D\omega^2}{m_n c^4} \sin^4 \theta - 1 \right) \right]^{-1};$$

$$\mu_2 = E^{-1} \delta(1 - \sigma^2) \left[ \left( \frac{C_n}{c} \sin \theta \right)^2 - \frac{1 - 2\sigma}{(1 - \sigma^2)} \right] \left[ \left( \frac{C_n}{c} \sin \theta \right)^2 - 1 \right]^{-1}.$$

Having carried out a numerical analysis of the bending vibrations of the shadow side of the plate, setting for concreteness –  $\delta = 1 \cdot 10^{-3} \text{ м}$ ;  $\eta = 0,1$ ;  $E = 71 \text{ ГМ}^2$  (Aluminium alloy D1 (0);  $\sigma = 0,31$ , then, the oscillations generated in the plate can be considered in terms of the amplitude and length of the bending wave [12] (Picture 5).

If on the y-axis we plot the value of the wavelength  $\lambda$  bending vibrations generated on the shadow side of the plate, and along the abscissa axis - the value  $\omega$  circular frequency, then the graph, for example, at an angle  $\theta = 0,985 \text{ рад}$  (56,25 degree) will take the form of a continuous curve of complex configuration, but symmetrical about the y-axis (Picture 5, curve 1) and having characteristic “bursts”. In between, at frequencies  $\omega = 200 \text{ с}^{-1}$  and  $\omega = 25 \cdot 10^3 \text{ с}^{-1}$ , the wavelength will decrease monotonically. It is obvious that the average power of the process is distributed unevenly over the frequencies  $\omega$  incident sound wave ranging from zero to  $40 \cdot 10^3 \text{ с}^{-1}$ . So, at frequencies  $\omega_c$  equal  $8,4 \cdot 10^3 \text{ с}^{-1}$ ,  $16,6 \cdot 10^3 \text{ с}^{-1}$ ,  $33,4 \cdot 10^3 \text{ с}^{-1}$  the spectrum shows a superposition of two modes of vibrations of different amplitudes and lengths (curves 2). The dotted line in fig. 5 shows the modulating wave lengths of the main bending vibrations. This phenomenon is observed at frequencies  $\omega_o$ , equal  $12,6 \cdot 10^3 \text{ с}^{-1}$ ,  $37,6 \cdot 10^3 \text{ с}^{-1}$  and corresponds to the passage of resonant regions [13; 14].



**Fig. 5. Change in flexural wave length**

With increasing angle  $\theta$  As the sound wave falls, the spectrum of flexural vibrations becomes more saturated, and their shape becomes more complex. The diagram, as it were, "shrinks" along the frequency axis. At frequency  $\omega = 0,4 \cdot 10^3 \text{ c}^{-1}$  the oscillation phase changes to  $\pi \text{ rad}$  [15; 16].

**Conclusions.** Thus, numerical analysis suggests that, other things being equal, in a diffuse field, the amplitude of the bending wave with increasing frequency  $\omega$  also decreases exponentially.

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