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DETERMINATION OF THE ORDER OF SINGULARITY OF A STRESS FIELD IN THE VICINITY OF ANGULAR POINT OF LINEAR PROBLEMS OF VISCOELASTICITY

Summary. *The method of determining the order of singularity of stress distribution in the vicinity of the angular point of the line of separation of the regions of inclusions of a flat viscoelastic body is presented.*

Key words: *flat viscoelastic body, inclusion circuit, angular point, singularity order, Airy stress function, viscoelastic operators*.

Introduction. For areas with piecewise-smooth boundaries, there are significant difficulties in determining the fields of displacement and stress near the breakpoints of the contours using the direct method of boundary elements. To solve this problem, various approaches were considered, in particular, the concept of independent multiple nodes [5], in the implementation of which at the stage of construction of discrete analogues of boundary integral equations introduced multiple nodes instead of one angular, as well as a method based on rounding angles [4]. All these methods have significant disadvantages, associated with either a significant increase in additional ratios or loss of accuracy, even at a short distance from the corner points, while for a number of problems you need to know the exact solution around the breakpoints of the body contours.

The goal of the work. Determining the order of the stress field singularity near the angular points of the contours of inclusions of flat viscoelastic piecewise homogeneous bodies.

Research methods and results. To take into account the influence of angular point type concentrators on the stress-strain state in a viscoelastic piecewise homogeneous body, the approach proposed in [1] was used to study the stress distribution in the vicinity of the angular point of the section of the cross-sectional areas of a composite elastic body that is in a state of flat deformation.

The study of the stress state of a composite viscoelastic body near the angular point of the line of separation of two regions was performed using Airy stress functions that satisfy the biharmonic equation, boundary conditions and continuity conditions of the respective components of the displacement vector and stress tensor at the line of separation of two regions [2]. In this case, the elastic constants of the materials according to [3] were replaced by viscoelastic operators.

According to [1], the solution of the biharmonic equation

$$\nabla^4 F_i = 0, \quad (i = 1, 2) \tag{1}$$

is written in the form:

$$F_i(r, \theta, t) = r^{s(t)+1}\Phi_i(s(t), \theta), \quad (i = 1, 2), \tag{2}$$

where Φ_i $(i = 1, 2)$ – Airy stress functions, $s(t) -$ parameter that depends on time, $r, \theta -$ geometric parameters of the vicinity the corner point of the dividing line of the two areas.

After substitution (2) in equation (1) we get the usual differential equation of the fourth order

$$\Phi_i^{(4)} + \left(2\left(s(t)\right)^2 + 2\right)\Phi_i^{(2)} + \left(\left(s(t)\right)^2 - 1\right)\Phi_i = 0, \ (i = 1, 2) \tag{3}$$

with boundary conditions on the line of separation

$$\Phi_1 = \Phi_2, \qquad \Phi'_1 = \Phi'_2 \tag{4}$$

and conjugation conditions for solutions

$$\frac{1+\tilde{\nu}_1}{\tilde{E}_1}[(1-\tilde{\nu}_1)\Phi''_1+(s(t)+1)(1-\tilde{\nu}_1-s(t)\tilde{\nu}_1)\Phi_1]= \\ =\frac{1+\tilde{\nu}_2}{\tilde{E}_2}[(1-\tilde{\nu}_2)\Phi''_2+(s(t)+1)(1-\tilde{\nu}_2-s(t)\tilde{\nu}_2)\Phi_2], \tag{5}$$

where $\tilde{E}_i$, $\tilde{\nu}_i$, $(i=1,2)$ – viscoelastic operators belonging to the class of resolvent operators [3, 6].

The general solution of equation (4) has the following form

$$\Phi_i(s(t),\theta)=A_i(t)\sin(s(t)+1)\theta+B_i(t)\cos(s(t)+1)\theta+ \\ +C_i(t)\sin(s(t)-1)\theta+D_i(t)\cos(s(t)-1)\theta, \qquad i=1,2.$$

Satisfying conditions (4), (5) for the branches $\theta=0$ and $\theta=\alpha$ of the line of separation, we obtain a system of linear algebraic equations for determining the unknown coefficients $A_i(t),\ B_i(t),\ C_i(t),\ D_i(t),\quad i=1,2.$

The existence of a nontrivial solution requires that the determinant of this system equal to zero:

$$\Delta(s(t),\tilde{\mu}_s,\tilde{m}_1,\tilde{m}_2)=0, \tag{6}$$

where $\tilde{\mu}_s=\dfrac{\tilde{E}_1}{\tilde{E}_2\left(\dfrac{1+\tilde{\nu}_2}{1+\tilde{\nu}_1}\right)}$, $\quad \tilde{m}_i=1-\tilde{\nu}_i, \qquad i=1,2.$

Solving equation (6), we set the value of $s(t)$, which is the root with the least positive real part of this transcendental equation. Therefore, the order of the singularity is equal to $|Res(t)-1|$.

Conclusions. The study of the peculiarity of the stress state in the flat case for a composite viscoelastic body near the angular point of the line of separation of two areas leads to the determination of the root with the smallest positive integer part of the transcendental equation depending on the viscoelastic parameters $\tilde{E}_i$, $\tilde{\nu}_i$, $(i=1,2)$ and angle θ.

The nature of the stress state at the apex of the angle of a composite flat body is determined by the type of boundary conditions, the viscoelastic characteristics of the material, the geometry of the areas and does not depend on the type of load.

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