#### Физико-математические науки

## **Karimov Vagif**

Candidate of Technical Sciences, Associate Professor of the Department of General and Applied Mathematics Azerbaijan State Oil and Industry University

# Керимов Вагиф Асад оглы

кандидат технических наук, доцент кафедры Общей и прикладной математики Азербайджанский государственный университет нефти и промышленности

## Karimova Sevinj

Senior Teacher of the Department of General and Applied Mathematics Azerbaijan State Oil and Industry University

## Керимова Севиндж Рафиг гызы

старший преподаватель кафедры Общей и прикладной математики Азербайджанский государственный университет нефти и промышленности

# ТНЕ MODEL OF CONTROL AND MANAGEMENT OF THE STATE OF THE ENVIRONMENT IN THE REGION МОДЕЛЬ КОНТРОЛЯ И УПРАВЛЕНИЯ СОСТОЯНИЕМ ОКРУЖАЮЩЕЙ СРЕДЫ В РЕГИОНЕ

**Summary.** The presented model is based on the consideration of a certain region as a combination of a certain number of areas. The ecological state of the region is estimated on the basis of negative and positive environmental factors in each area. The model is based on the imitation of the interaction of environmental

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and economic processes. The resulting model can be used in the management of the ecological-economic system of the region.

*Key words:* simulation models, ecological economic system, global ecological economic models, anthropogenic impacts, environmental pollution, maximum permissible value.

Аннотация. Представленная модель построена на основе рассмотрения некоторого региона как совокупности определенного количества районов. Экологическое состояние региона оценивается на основе отрицательных и положительных факторов влияния на окружающую среду в каждом районе. Модель построена на основе имитации взаимодействия экологических и экономических процессов. Полученную модель можно использовать в процессе управления экологоэкономической системой региона.

Ключевые слова: имитационные модели, эколого-экономическая система, глобальные эколого-экономические модели, антропогенные воздействия, загрязнение окружающей среды, предельно-допустимое значение.

Since the middle of the 20th century, mathematical methods have been widely used in the study of environmental problems [1]. In recent years, the problem of confrontation between man and nature has become especially urgent [2-3].

The task of developing of the model in which at each moment of time some part of the studied ecological-economic system is identified as an object of intensified pollution was set in the presented article. The idea is to provide a regulating effect at the moment to this particular subsystem.

Considered tasks are formulated on the basis of the initial information on the state of the environment, during the collection of which, as a rule, field measurements are performed, aerospace surveys are used, and various methods of monitoring over environmental objects are used [4]. If there is sufficient information, then the ecological-economic system under consideration can be described by a model

$$\langle s, f(s), f_0(s) \rangle \tag{1}$$

Here *s* is a generalized indicator of the considered system. Depending on the context of presentation in one case, *s* will be used to designate the system itself, in another case, it will be used when we evaluate a certain quantitative indicator of the system. The area we are considering, the number of people living here, etc., can be such an indicator. f(s) is indicator of environmental stress of this system;  $f_0(s)$  is maximum permissible value of the indicator f(s). The value of  $f_0(s)$  is established by various sanitary and regulatory methods. Prior popularity of this indicator is allowed

$$f(s) > f_0(s) \tag{2}$$

This inequality indicates that in this system there is an ecologically stressful state. Some measures are required to relieve or reduce this tension. As a result of these measures the following condition would be satisfied:

$$f(s) \le f_0(s) \tag{3}$$

Suppose that using some method, this system can be divided into a finite number of subsystems  $s_i$ , i = 1,...,n. We associate with each parameter  $s_i$  some values  $f(s_i)$ ,  $f_0(s_i)$  similar to f(s) and  $f_0(s)$ .

Task 1. It is necessary to develop a method for splitting a set  $\{s_i\}$  into such subsets  $\{s_p^1\}$  and  $\{s_q^2\}$  for which the following conditions would be satisfied

$$f(s_{p}^{1}) \le f_{0}(s_{p}^{1}) \tag{4}$$

$$f(s_q^2) > f_0(s_q^2)$$
 (5)

$$p = 1, n_1; q = 1, n_2; n_1 + n_2 = n_1$$

It is clear that the constructed model identifies the subsystems from  $s_p^1$  ecologically clean (non-polluting), and from  $s_q^2$  - ecologically tense.

Task 2. Develop a method for ranking subsystems  $s_q^2$  by the following inequality

$$f(s_q^3) \le f(s_{q+1}^3)$$
 (6)

where

$$s_q^3 \in \{s_q^2\}, \quad q = \overline{1, n_2 - 1}.$$

The sequence  $s_q^3$  is constructed by rearranging the elements of the sequence  $s_q^2$ .

The constructed model makes it possible to identify the subsystem  $s_{n_2}^3$  as ecologically most stressful, subsystem  $s_{n_2-1}^3$  as following in level of environmental stress, etc.

Task 3. Find a method for constructing numbers  $\alpha_q$  from the interval (0;1) so that the following condition is satisfied

$$\alpha_q f(s_q^3) \le f_0(s_q^3) \tag{7}$$

where

$$q = n_2, n_2 - 1, \dots, n_2 - n_0; \quad 0 \le n_0 \le n_2 - 1.$$

The last task is directed to the fact that, starting from the most highly ecologically tense subsystem and ending on some  $s_{n_2-n_0}^3$ , they are subjected to such influence or control, at which their environmental tensions were removed. In the worst case, only one subsystem  $s_{n_2}^3$  is exposed to such an impact, and in the best case, all elements from the set  $\{s_q^3\}$  are exposed to such an impact. As a result of solving task 3, it becomes possible to overestimate the parameter f(s) and establish how much the environmental situation in the considered system has changed towards improvement.

**Description of the model.** In this paper, the concept and mathematical model for reducing environmental tensions, which is implemented on the basis of statistical data about the industrial region under study, is proposed.

Factors affecting the environment can be divided into 2 groups.

Group I - negative factors. This is the number of plants and factories; the length of railways, roads; population number; area of contaminated areas, landfills, etc. Group II-positive for the environment factors. This is the area of pasture, greenery; the area of forests, parks, lawns, unused areas, recreational facilities, etc.

The high value of the parameters  $x_{ij}$  in the first group means a high environmental tension of the current situation in the study area. Here  $x_{ij}$  is the value of *i* -th negative factor for *j* -th area;  $i = \overline{1,n}$ ;  $j = \overline{1,m}$ . Let us introduce the parameters  $f_j$  determining the level of the total impact of negative factors on the state of the environment in each *j* -th area :

$$f_j = \sum_{i=1}^n a_i x_{ij}, \quad j = \overline{1, m}$$
(8)

Here  $a_i$  are weights of negative factors,  $a_i \ge 0$ ,  $\sum a_i = 1$ 

The total negative impact on the environment of the industrial region under consideration can be assessed by the formula:

$$F = \sum_{j=1}^{m} f_j \tag{9}$$

The relative impact on the environment for each area with number *j* can be estimated using the formula:

$$\widetilde{f}_j = \frac{f_j}{F} \tag{10}$$

Let  $y_{ij}$  be the amount of positive factor with the number *i* for the area with the number j ( $i = \overline{1,k}; j = \overline{1,m}$ ).

You can build similar formulas to assess the impact of positive factors on the environment of the districts and the region as a whole.

$$g_{j} = \sum_{i=1}^{k} b_{i} y_{ij}$$
(11)

$$G = \sum_{j=1}^{m} g_j \tag{12}$$

$$\widetilde{g}_j = \frac{g_j}{G} \tag{13}$$

Here  $b_i$  are weight coefficients of positive factors ,  $b_i \ge 0$ ,  $\sum b_i = 1$ ,  $g_j - is$ the estimate of the total impact of positive factors on the environment of the *j*-th district; *G* is assessment of positive factors on the environment of the whole region;  $\tilde{g}_i$  is relative assessment of positive factors on the environment in the area with the number *j*.

To identify areas where the ecological-economic balance is disturbed, it is proposed to use the parameter:

$$\beta_j = \frac{\widetilde{f}_j}{\widetilde{g}_j} \tag{14}$$

We denote the set of areas by  $J = \{1, ..., m\}$ . This set can be divided into two subsets :  $J = J_1 \cup J_2$ ,  $J_1 \cap J_2 = \emptyset$ ,  $J_1 = \{j \mid \beta_j > 1\}$ ,  $J_2 = \{j \mid \beta_j \le 1\}$ .

Definition 1. Assume that the system described at time  $t_1$  by  $\langle t_1, J_1, J_2, \beta_j \rangle$ , in time  $t_2$  is described by  $\langle t_2, J'_1, J'_2, \beta'_j \rangle$ . Then, if the conditions  $J'_1 = J_1$  and  $\beta'_j \langle \beta_j (j \in J_1)$  are satisfied, then we will say that during the time  $\Delta t = t_2 - t_1$  in the areas of the sets  $J_1$ , the ecological-economic balance has normalized.

However, in the case of the transition of the system from time  $t_1$  to  $t_2$ , some areas that previously belong to set  $J_2$  may move in set  $J_1$ , as a result of which it turns out that  $J'_1 \neq J_1$ .

Therefore, it is necessary to consider such a method of normalizing the ecological situation, in which all areas are taken into account, not individual areas, i.e. whole region.

If in a certain area the values of negative factors are reduced, and positive factors are increased by certain percentages, and all other factors are not changed, then the ecological-economic balance is normalized in this area.

Definition 2. If, in the previous definition, conditions  $\beta'_j < \beta_j$   $(j \in J_1)$  are replaced by  $\beta' \leq \beta_j$ , then we will say that during time  $\Delta t$  in areas from the set  $J_1$ , the ecological-economic balance is not disturbed.

As you know, environmentally harmful areas can be determined by the amount of hazardous waste per person, or per unit area of the district.

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In addition, there is a formula that determines the most ecologically harmful region on the basis of the sum of the two indicators:

$$\Phi_j = \frac{\varphi_j}{N_j} + \frac{\varphi_j}{S_j} \tag{15}$$

In the formula,  $\varphi_j, N_j, S_j$  –are the amount of pollution, population and area of the district with number *j*.

If for some area the inequality

$$\Phi_i < \Phi_0 \tag{16}$$

is satisfied, then according to this indicator, this area is not considered environmentally harmful.

Here, the parameter  $\Phi_0$  is the maximum permissible value of the parameter  $\Phi_j$ , which is estimated by sanitary and hygienic standards with the participation of experts in this field [4].

Consequently, the areas for which the inequality

$$\Phi_j \ge \Phi_0 \tag{17}$$

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is satisfied are attracted to the model of ecological-economic balance. Consider the dispersion:

$$D = \frac{\sum_{j=1}^{m} \left(\widetilde{f}_{j} - \widetilde{g}_{j}\right)^{2}}{m}$$
(18)

If the value of the indicator  $\Phi_0$  is unknown, or if it cannot be achieved, the developed methodology can be used to minimize the value of the indicator *D*, the minimum value of which is 0. D = 0 means that the share in the pollution of the environment of each area is equal to its share in the purification of the environment. Differences between the share in the pollution and purification of the considered areas are different due to their heterogeneity. Therefore, the requirement to achieve D = 0 is not practical. It is practical to minimize the value of the parameter *D* to a certain level. How to determine the desired lower level of the parameter *D*? The answer to this question can be given using computational experiment.

Let for the current time all positive and negative factors for the environment be known, as well as their values. We are interested in how much you can reduce the values of negative factors, as well as increase the values of positive factors in order to achieve  $D_{\min} = D_0$ . This task can be solved by a survey conducted among the leaders and responsible persons of administrative districts. In view of these considerations, a practical condition for ecological and economic equilibrium is  $|D - D_0| \le \varepsilon$ , where  $\varepsilon$  is some small number.

Consider a case where all negative factors  $x_{ij}$  are reduced at the same scale  $k_1$ . Positive factors, on the contrary, increase in scale  $k_2$ , at which the factors  $x_{ij}$  and  $y_{ij}$  take on new values, which we denote by  $x'_{ij}$  and  $y'_{ij}$ 

$$x'_{ij} = k_1 x_{ij}$$
(19)

$$y'_{ij} = k_2 x_{ij}$$
 (20)

In the last formulas,  $0 < k_1 < 1, k_2 > 1$ .

Theorem 1. As a result of changing the parameters  $x_{ij}$  and  $y_{ij}$ , carried out according to formulas (19) and (20), the values of the parameters  $\beta_j$  will not change.

Proof: New parameter values  $f_j, F, \tilde{f}_j, g_j, G, \tilde{g}_j, \beta_j$  are denoted accordingly  $f'_j, F', \tilde{f}'_j, g'_j, G', \tilde{g}'_j, \beta'_j$ .

We obtain the following relations:

$$f'_{j} = \sum_{i=1}^{n} a_{i}k_{1}x_{ij} = k_{1}f_{j}$$
$$F' = \sum_{j=1}^{m} f'_{j} = \sum_{j=1}^{m} k_{1}f_{j} = k_{1}F$$
$$\tilde{f}'_{j} = \frac{f'_{j}}{F'} = \frac{k_{1}f_{j}}{k_{1}F} = \tilde{f}_{j}$$

The equality  $\tilde{g}'_j = \tilde{g}_j$  is proved in a similar way, from which it follows that  $\beta'_j = \beta_j$ .

This fact allows us to conclude that in the studied region in order to regulate the ecological-economic balance it is necessary to take measures for those areas for which the parameter  $\beta_i$  proves the imbalance.

Theorem 2. If in a certain area j of the studied system, the values of negative factors are reduced at a certain scale, and all other factors do not change, then the value of the ecological-economic balance of this region will decrease.

Proof: Let  $f'_j = k_1 f$ . Since  $0 < k_1 < 1$  then  $k_1$  can be represented as

$$k_1 = \left(1 - \frac{\alpha_1}{100}\right), \quad 0 < \alpha < 100.$$

It is clear that as a result of the reduction of the parameters  $x_{ij}$ , the value of the parameter *F* is also reduced:

$$1 - \frac{\alpha_2}{100}, \quad \alpha_2 < \alpha_1$$

Given the above, you can get the following results:

 $\beta'_{j} = f'_{j} \cdot G / (g_{j} \cdot F') = G \cdot (100 - \alpha_{1}) f_{j} / [g_{j} \cdot (100 - \alpha_{2})F] = \beta_{j} (100 - \alpha_{1}) / (100 - \alpha_{2}).$ It follows that  $\beta'_{j} \le \beta_{j}$ .

Similarly, we can prove Theorem 3.

Theorem 3. If in a certain area j of the studied system the values of positive factors increase on a certain scale, and all other factors do not change, then the value of the ecological-economic balance of the given area will decrease.

Conclusion. If in a certain area j the values of positive factors increase, and the values of negative factors decrease, then the value of the parameter  $\beta_j$ decreases.

Analytical conclusions, as well as experiments performed on a computer, show that a decrease of the parameter  $\beta_{j_0}$  can lead to an increase of the parameters  $\beta_j$ , for  $j \neq j_0$ . This explains the possibility of infinite control over the state of the environment with the help of the developed model.

The solution algorithm consists of the following steps:

- 1. Beginning;
- 2. Enter values  $x_{ij}$ ,  $y_{ij}$ ,  $a_i$ ,  $b_i$ ,  $\varepsilon$ ;
- 3. Estimate of parameters  $f_j$ , F,  $\tilde{f}_j$ ,  $g_j$ , G,  $\tilde{g}_j$ ;
- 4. Calculating of dispersion D by the formula (18);
- 5. If  $|D D_0| \le \varepsilon$ , then print "the ecological-economic balance is not disturbed", end, otherwise proceed to the next step ;
- 6. Determine  $\max_{j} \{ \tilde{f}_j \tilde{g}_j \} = \tilde{f}_{j_0} \tilde{g}_{j_0}$ ;
- 7. In the area with the number  $j_0$ , carry out activities of type "A" or "B" or "C" and go to step 2; where "A" is the event to reduce the values of the parameters  $x_{j_0}$ , "B" is the event to increase the values of the parameters  $y_{j_0}$ , "C" = "A" & "B".

The developed model of control over the state of the environment can be used both when the value of  $\Phi_0$  is known, or when it is not known, or when it cannot be achieved. The fact is that very often the values of the parameter  $\Phi_0$  for different countries, sometimes even different regions of one large country, are evaluated differently [4]. This circumstance raises doubts about the objectivity of such an assessment. Therefore, alternative assessment methods are always helpful. In the case of not using the value of  $\Phi_0$  in the calculations, you can use the value of  $D_0$ , that is, the minimum value of the parameter D. Maintaining the condition  $|D - D_0| \le \varepsilon$  directs the functioning of the system to a rhythm at which the ratios of participatory shares in pollution and environmental clean-up in all areas would be approximately equal.

Thus, the presented model allows realizing a cybernetic system for controlling and managing of the state of the environment [5-6].

## References

- Volterra V. Mathematical Theory of Struggle for Existence. M.: Science, Moscow, 1976. 288 p.
- 2. Jay Wright Forrester, World Dynamics. Wriht-Allen Press, 1971. 142 p.
- Medouz D.H., Medouz D.L., Randers Y. Outside growth. Moscow: Progress - Pangaea, 1994. 304 p.
- 4. Izrael Yu. A. Ecology and Control of the Natural Environment. Kluwer Academic Publishers, 1992. 375 p.
- Riznichenko G. Yu. Mathematical models in biophysics and ecology. M-Izhevsk: Institute of Computer Sciences, 2003. 184 p.
- Zaslavsky B.G., Poluektov R.A. Management of environmental systems.
   M: Science, 1988. 296 p. (Theory and methods of systems analysis).