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МЕТОД РОЗВ'ЯЗУВАННЯ СИСТЕМИ РІВНЯНЬ ВАРІАНТА МАТЕМАТИЧНОЇ ТЕОРІЇ НЕ ТОНКИХ ПОЛОГИХ ОБОЛОНОК

МЕТОД РЕШЕНИЯ СИСТЕМЫ УРАВНЕНИЙ ВАРИАНТА МАТЕМАТИЧЕСКОЙ ТЕОРИИ НЕТОНКИХ ПОЛОГИХ ОБОЛОЧЕК

METHOD OF SOLUTION EQUATION SYSTEM WITHIN THE VARIANT OF MATHEMATICAL THEORY OF NON-THIN SHALLOW SHELLS

***Анотація**: В статті запропоновано метод, який дає можливість звести розв'язувальну систему неоднорідних диференціальних рівнянь із частинними похідними шістнадцятого порядку варіанта математичної теорії не тонких пологих оболонок до диференціальних рівнянь другого і* *четвертого порядків. Використовується метод збурень геометричних параметрів і операторний метод.*

***Ключові слова**: варіант математичної теорії, не тонка полога оболонка, система неоднорідних диференціальних рівнянь із частинними похідними, метод збурень, оператор.*

***Аннотация:** В статье предложен метод, который дает возможность привести разрешающую систему неоднородных дифференциальных уравнений в частных производных шестнадцатого порядка варианта математической теории не тонких пологих оболочек к дифференциальным уравнениям второго и четвертого порядков. Используется метод возмущения геометрических параметров и операторный метод.*

***Ключевые слова**: вариант математической теории, не тонкая пологая оболочка, система неоднородных дифференциальных уравнений в частных производных, метод возмущений, оператор.*

***Summary:** In this paper, method is offered enabling reduction of resolving system of heterogeneous partial-derivative differential equations of the sixteenth order within the variant of mathematical theory of non-thin shallow shells to the differential equations of the second and fourth orders. A method is used of geometrical parameters perturbation and symbolical method.*

***Keywords:** variant of mathematical theory, non-thin shallow shell, system of heterogeneous partial-derivative differential equations, method of perturbations, operator.*

Introduction. In the case of steep gradient of the stress-strain state (SSS) variation, classical theories of thin and non-thin plates and shells under conditions of local loads, existence of holes and sharp variation of mechanical and geometrical parameters provide unsatisfactory results, which could substantially differ from exact ones. Non-classical theories based on various hypotheses and assumptions for the very large class of boundary problems are also basically unable to describe SSS of plates and shells with any high accuracy, since SSS components are represented as a small number of summands. In addition, obtained differential equation (DE) systems are of low order. Studies on various theories are reviewed in [1, P.3-32; 2, P. 22-57].

Approach offered in [3, P. 51-58] for calculation of plate under the skew-symmetric loading in the first two approximations was generalized by author in a number of studies for the physically linear and nonlinear, solid and laminated non-thin plates and shallow shells (reviewed in [4, P. 21-30]). The developed variant of mathematical theory of non-thin elastic plates and shallow shells [4, P. 21-30] is based on 1) representation of all SSS components as three-dimensional functions represented by the Legendre polinomials series depending on the transverse coordinate and satisfying exactly to the boundary conditions on the face planes (surfaces); 2) the use of the variational Reissner's principle [5, P. 90-95] for reduction of three-dimensional boundary problem for plates and shells of arbitrary constant thickness to two-dimensional one; 3) the use of coupled equations. As a result, boundary problem is reduced to the solution of the heterogeneous partial-derivative differential equation (DE) system with respect to constituents of motion components. The DE system order and boundary problem solution accuracy are determined by the number of terms retained in series.

The point to be emphasized is that SSS components presentation as series basically enables its determination with any high accuracy. However, that results in increased mathematical complexity, since order of the partial-derivative DE systems is increased with respect to the sought-for functions. Consequently, a need arises in development of mathematical methods of high order heterogeneous DE systems reduction to low order equations, in particular, to second and fourth order equations.

1. Problem formulation. From the perspective of three-dimensional elasticity theory, transversely isotropic shallow bicurved shell of arbitrary constant thickness h is considered in the Cartesian coordinate system $Oxyz$. Oz axis, of which origin is in the median shell surface, is directed toward its convexity. Skew-symmetric loading is applied to the shell. Boundary conditions on the face surfaces are as follows:

$$\sigma_z(z=\pm h/2)=\mp q(x,y)/2; \;\; \sigma_{xz}(z=\pm h/2)=\sigma_{yz}(z=\pm h/2)=0,$$

where $q(x,y)$ – transverse loading intensity.

Boundary conditions on the lateral surface, which is assumed to be normal to the median shell surface, can be specified in stresses, motions or in stresses and motions (mixed problem).

The next two paragraphs provide outline of basic relationships and equations of the considered variant of mathematical theory previously obtained by author.

2. Approximation of SSS components. Constituents of motion components $U(x,y,z)$, $V(x,y,z)$, $W(x,y,z)$ are represented by series with the use of the Legendre polinomials:

$$U(x,y,z)=\sum_{k=0}^{\infty}P_k(\frac{2z}{h})u_k(x,y)(U,V;u,v), W(x,y,z)=\sum_{k=1}^{\infty}P_{k-1}(\frac{2z}{h})w_k(x,y), \quad (1)$$

where $P_k(2z/h)$- Legendre polinomials, $u_k(x,y)$, $v_k(x,y)$, $w_k(x,y)$ – sought-for constituents of motion components.

For the transversely isotropic non-thin shallow shell, of which isotropy plane is parallel to the xOy plane at each point of space, dependencies between strains and motions are represented as the following series:

$$\varepsilon_x = \sum_{j=0}^{\infty} \varepsilon_{xj}\,,\ (x,y);\ \ \varepsilon_z = \sum_{j=1}^{\infty} \varepsilon_{zj}\,;\ \gamma_{yx} = \sum_{j=0}^{\infty} \gamma_{yxj}\,;\ \gamma_{xz} = \sum_{j=1}^{\infty} \gamma_{xzj}\,,\ (x,y), \tag{2}$$

where

$$\varepsilon_{xj}(x,y,z) = P_j\left(\frac{\partial u_j(x,y)}{\partial x} + k_1 w_{j+1}(x,y)\right),\ (x,y;u\to v;k_1\to k_2);$$

$$\varepsilon_{zj}(x,y,z) = P_j'\, w_{j+1}(x,y);\ \gamma_{yxj}(x,y,z) = P_j\left(\frac{\partial u_j(x,y)}{\partial y} + \frac{\partial v_j(x,y)}{\partial x}\right);$$

$$\gamma_{xzj}(x,y,z) = P_j\,\frac{\partial w_{j+1}(x,y)}{\partial x} + P_j' u_j(x,y) - k_1' P_j u_j(x,y),\ (x,y;u\to v;k_1'\to k_2');$$

$$(x,y;\,U\to V;\,k_1'\to k_2'),\ (k_i = 1/R_i,\ k_i' = k_i,\ i=1,2;\ k_{1v} = k_1 + k_2 v;$$

$$k_{2v} = k_2 + k_1 v).$$

Here $k_1,\ k_2$ are principal curvatures, and $R_1,\ R_2$ are main curvature radii of median shell surface. Since the shell is non-thin one, expressions for the transverse angular strains γ_{xz}, γ_{yz} take into account the tangential displacement components by means of summands containing k_i' and k_2' (these are ignored in classical theory).

Stresses in shell are also represented as series:

$$\sigma_{xz}(x,y,z) = \sum_{i=0}^{\infty} P_i t_{xi};\ \sigma_{yz}(x,y,z) = \sum_{i=0}^{\infty} P_i t_{yi}\,,\ \sigma_z(x,y,z) = \sum_{i=0}^{\infty} P_i s_{zi}\,; \tag{3}$$

$$\sigma_x(x,y,z) = \sum_{i=0}^{\infty} P_i s_{xi}\ \ (\sigma_x\to\sigma_y;\ s_{xi}\to s_{yi});\ \sigma_{xy}(x,y,z) = \sum_{i=0}^{\infty} P_i t_{yxi},$$

where $t_{xi},\dots,t_{yxi}$ depend on $u_k(x,y), v_k(x,y), w_k(x,y)$ и and their derivatives [4, P.21-30].

If we assume curvatures k_1 and k_2, k_1' and k_2' in (2) and (3) to be equal to zero, we obtain respective dependencies for plate.

3. Initial differential equation system and its transformation. Let us consider the skew-symmetric transverse loading as the approximation $k = 0,1,3$ (constituents with subscripts 0, 1, 3, i.e. $u_0,u_1,u_3;\ v_0,v_1,v_3;$ are only taken into account in series (1) for tangential components of motion $U(x, y, z)$ and $V(x, y, z)$; then constituents w_1 and w_3 are only taken into account for the transverse motion $W(x, y, z)$). Resolving equations are represented by the following heterogeneous partial-derivative DE system [6, P. 131-139]:

$$\gamma_{111}u_{0,xx} + \gamma_{112}u_{0,yy} + \gamma_{121}v_{0,xy} + k_{1w1}w_{1,x} + k_{1w3}w_{3,x} = 0; \tag{4}$$

$$\gamma_{121}u_{0,xy} + \gamma_{112}v_{0,xx} + \gamma_{111}v_{0,yy} + k_{2w1}w_{1,y} + k_{2w3}w_{3,y} = 0;$$

$$\beta_{113}u_1 + \beta_{133}u_3 + \beta_{111}\varphi_{1,x} + \beta_{112}\psi_{1,y} + \beta_{131}\varphi_{3,x} + \beta_{151}w_{1,x} + \beta_{161}w_{3,x} = \beta_{u1}q_{,x};$$

$$\beta_{113}v_1 + \beta_{133}v_3 + \beta_{111}\varphi_{1,y} - \beta_{112}\psi_{1,x} + \beta_{131}\varphi_{3,y} + \beta_{151}w_{1,y} + \beta_{161}w_{3,y} = \beta_{u1}q_{,y};$$

$$\beta_{133}u_1 + \beta_{333}u_3 + \beta_{131}\varphi_{1,x} + \beta_{331}\varphi_{3,x} + \beta_{332}\psi_{3,y} + \beta_{351}w_{1,x} + \beta_{361}w_{3,x} = \beta_{u3}q_{,x};$$

$$\beta_{133}v_1 + \beta_{333}v_3 + \beta_{131}\varphi_{1,y} + \beta_{331}\varphi_{3,y} - \beta_{332}\psi_{3,x} + \beta_{351}w_{1,y} + \beta_{361}w_{3,y} = \beta_{u3}q_{,y};$$

$$k_{1w1}u_{0,x} + k_{2w1}v_{0,y} + \beta_{151}\varphi_1 + \beta_{351}\varphi_3 + (\beta_{551}\nabla^2 + r_{1w1})w_1 + (\beta_{561}\nabla^2 + r_{1w3})w_3 = \beta_{w1}q;$$

$$k_{1w3}u_{0,x} + k_{2w3}v_{0,y} + \beta_{161}\varphi_1 + \beta_{361}\varphi_3 + (\beta_{561}\nabla^2 + r_{1w3})w_1 + \\ + (\beta_{661}\nabla^2 + \beta_{663} + r_{3w3})w_3 = \beta_{w3}q,$$

where $\varphi_i(x, y) = u_{i,x} + v_{i,y}$, $\psi_i(x, y) = u_{i,y} - v_{i,x}$, ∇^2 – Laplacian operator, γ, β, k, r with subscripts are mechanical and geometrical parameters (MGPs) determined by mechanical and geometrical shell constants. Please note that subscripted γ and β constants are independent of curvature, i.e., the same as those for plate. Curvatures are only included in subscripted k and r MGPs. Therefore, if we assume the latter to be equal to zero, then the system (4) represents the resolving equations for the transversely isotropic plates (first two equations describe the flat problem, and last six ones describe problem of bending).

System (4) is reduced to two systems by means of mathematical transformations.

One of systems, namely, homogeneous fourth order system with respect to two vortex functions $\psi_1(x, y)$ and $\psi_3(x, y)$, describes a vortex edge effect (equations for plates are similar ones):

$$\begin{aligned} &\beta_{113}\psi_1 + \beta_{112}\nabla^2\psi_1 + \beta_{133}\psi_3 = 0, \\ &\beta_{133}\psi_1 + \beta_{332}\nabla^2\psi_3 + \beta_{333}\psi_3 = 0, \end{aligned} \tag{5}$$

Other system, namely, twelfth order coupled heterogeneous DE system with respect to u_0, v_0, w_1, w_3 constituents (let us denote these as basic ones), describes internal SSS with the potential edge effect:

$$P_{iu0}u_o + P_{iv0}v_o + P_{iw1}w_1 + P_{iw3}w_3 = P_{iq}q\,, (i = 1, 2, 3, 4), \tag{6}$$

where subscripted P – differential operators containing MGP. The other constituents of motion components are represented through the basic constituents from the third–sixth equations, in which $\varphi_1(x, y)$ and $\varphi_3(x, y)$ are expressed from seventh–eighth equations.

4. Method of perturbations. Resolving equations. We offer the method allowing reducing of mathematical complexities of solution of main equations (5) and (6). Let us introduce small parameter $\varepsilon = h/(R_1 + R_2)$. Then subscripted k and r MGPs are represented as follows:

$$k_{1w1} = K_{1w1}\,\varepsilon,\ k_{1w3} = K_{1w3}\,\varepsilon,\ k_{2w1} = K_{2w1}\varepsilon,\ k_{2w3} = K_{2w3}\,\varepsilon, \tag{7}$$

$$r_{1w1} = R_{1w1}\varepsilon^2,\ r_{1w3} = R_{1w3}\varepsilon^2,\ r_{3w3} = R_{3w3}\varepsilon^2,$$

where $K_{1w1},\ldots,R_{3w3}$ are final constant values independent of parameter ε.

The solution of the homogeneous DE system (5) is not difficult. It is reduced to the solution of two Helmholtz equations.

We solve the system (6) by the method of perturbations of geometrical shell parameters followed by the use of operator method for the solution of the obtained equations in every approximation.

We represent basic constituents of motion components as series in terms of the small geometrical parameter ε.

$$u_0(x,y)=\sum_{i=0}^{\infty}\varepsilon^i\,u_{0i}(x,y),\ (u,v);\ \ w_1(x,y)=\sum_{i=0}^{\infty}\varepsilon^i\,w_{1i}(x,y),\ (w_1,w_3)\,, \tag{8}$$

where $u_{0i}(x,y),\ v_{0i}(x,y),\ w_{1i}(x,y), w_{3i}\,(x,y)$ – sought-for functions.

Constituents of stress components, other constituents of motion components and lateral surface boundary conditions are also expanded into similar series (8).

By the asymptotic splitting of the DE system (6) taking into account (7) and (8), we obtain following two DE systems with respect to $u_{0i}\,, v_{0i}$ and $w_{1i}\,, w_{3i}$:

In the zero-order approximation:

homogeneous system of 4th order

$$\begin{aligned} M_{11}u_{00}+M_{12}v_{00}&=0;\\ M_{12}u_{00}+M_{22}v_{00}&=0 \end{aligned} \tag{9}$$

and heterogeneous system of 8th order

$$\begin{aligned} \Pi_{11}w_{10}+\Pi_{13}w_{30}&=\Pi_{1q}q;\\ \Pi_{31}w_{10}+\Pi_{33}w_{30}&=\Pi_{3q}q, \end{aligned} \tag{10}$$

where $M_{11},\ldots,M_{22}$, $\Pi_{1q}\,,\Pi_{3q}$– known differential operators of 2nd order; $\Pi_{11}\,,\ldots,\Pi_{33}$ – those of 4th order.

In the subsequent approximations $(i=1,2,\ldots)$, we also obtain two systems:

heterogeneous system of 4th order

$$\begin{aligned} M_{11}u_{0i}+M_{12}v_{0i}&=Q_{1(i-1)};\\ M_{12}u_{0i}+M_{22}v_{0i}&=Q_{2(i-1)} \end{aligned} \tag{11}$$

and heterogeneous system of 8th order

$$\begin{aligned} \Pi_{11}w_{1i}+\Pi_{13}w_{3i}&=P_{1(i-1)};\\ \Pi_{31}w_{1i}+\Pi_{33}w_{3i}&=P_{3(i-1)}. \end{aligned} \tag{12}$$

Right-hand sides of equations (11) depend on the solutions of previous $(i-1)^{\text{th}}$ approximation of the system (12), and right-hand sides of equations (12) depend on the solutions of previous $(i-1)^{\text{th}}$ approximation of the system (11) and the solutions of $(i-2)^{\text{th}}$ approximation of the system (12).

The system (9) corresponds to the flat problem of elasticity theory for plate, and (10) corresponds to the problem of plate bending. Systems (11) and (12) specify the solutions of flat problem and problem of bending, respectively.

When using this method of perturbations, lateral surface boundary conditions in the zero-order approximation would be generally heterogeneous, and would be homogeneous in the subsequent approximations.

So, boundary problem for non-thin transversely isotropic shell in the considered approximation $(k=0,1,3)$ is reduced by the method of perturbations to the following resolving equations: homogeneous and heterogeneous systems of 4^{th} order and two heterogeneous systems of 8^{th} order.

5. Reduction of the systems (9) – (12) to equations of 2^{nd} and 4^{th} orders. Let us consider the heterogeneous system of 8^{th} order (systems of (10) and (12) type). These systems are structurally similar and only differ in right-hand sides. For convenience, we write the system of (10) (or (12)) type as follows:

$$\begin{aligned} \Pi_{11}w_1 + \Pi_{13}w_3 &= f_1; \\ \Pi_{31}w_1 + \Pi_{33}w_3 &= f_3, \end{aligned} \tag{13}$$

where $f_1 = f_1(x,y), f_3 = f_3(x,y)$ – known functions.

The system of (13) type is reduced by the operator method to following two heterogeneous DE of 8^{th} order

$$D_1D_2D_3D_4\,F_k(x,y) = f_i\,(x,y)\ \ (i=1,3)\,, \tag{14}$$

where D_i – differential operators of 2^{nd} order:

$$D_1 = \nabla^2,\ D_2 = \nabla^2,\ D_3 = (\nabla^2 - a_3),\ D_4 = (\nabla^2 - a_4)\,. \tag{15}$$

Here a_3, a_4 – some constants being roots of characteristic equation (these can be complex ones).

The general solution of the system (13) is expressed through the general solution $F_0(x, y)$ of the homogeneous equation corresponding to (14) and two partial solutions $F_{1r}(x, y)$ and $F_{3r}(x, y)$ of heterogeneous equations (14):

$$w_1(x, y) = \Pi_{33}(F_{10} + F_{1r}) - \Pi_{13}F_{3r}; \; w_3(x, y) = -\Pi_{31}(F_{10} + F_{1r}) + \Pi_{11}F_{3r}, \qquad (16)$$

where

$$F_{10}(x, y) = F_{1B}(x, y) + F_{1\Pi 1}(x, y) + F_{1\Pi 2}(x, y). \qquad (17)$$

Here F_{1B} – general solution of bi-harmonic equation $\nabla^4 F_{1B} = 0$, $F_{1\Pi 1}$ and $F_{1\Pi 2}$ – general solutions of two differential Helmholtz equations:

$$(\nabla^2 - a_3)F_{1\Pi 1}(x, y) = 0, \; (\nabla^2 - a_4)F_{1\Pi 2}(x, y) = 0. \qquad (18)$$

We then obtain partial solutions of equations (14) by operator method through the partial solutions of heterogeneous equations of 2nd and 4th orders.

Let us consider equation of (14) type:

$$D_1 D_2 D_3 D_4 \, F(x, y) = f(x, y), \qquad (19)$$

where $f(x, y)$ – known function, and $F(x, y)$ – sought-for function.

We represent partial solution $F_r(x, y)$ of equation (19) as follows:

$$F_r(x, y) = \frac{1}{D_1 D_2 D_3 D_4} f(x, y), \qquad (20)$$

where $1/(D_1 D_2 D_3 D_4)$ – inverse operator.

Then suppose $F_{ir}(x, y), (i = 1, \ldots, 5)$ are partial solutions of heterogeneous equations:

$$D_i F_i(x, y) = f(x, y) \;\; (i = 1, \ldots, 4), \; D_1 D_2 F_5(x, y) = f(x, y).$$

These solutions can be represented through inverse operators as follows

$$F_{ir}(x, y) = \frac{1}{D_i} f(x, y), \; F_{5r}(x, y) = \frac{1}{D_1 D_2} f(x, y). \qquad (21)$$

We transform right-hand side of equation (20) taking into account commutativity and associativity of operators:

$$\frac{1}{D_1D_2D_3D_4}f(x,y)=\frac{1}{(D_1D_3)(D_2D_4)}f=$$
$$=\frac{1}{(D_1-D_3)}\left(\frac{1}{D_3}-\frac{1}{D_1}\right)\frac{1}{(D_2-D_4)}\left(\frac{1}{D_4}-\frac{1}{D_2}\right)f.$$

With consideration of (15), we obtain

$$F_r=\frac{1}{a_3}\left(\frac{1}{D_3}-\frac{1}{D_1}\right)\frac{1}{a_4}\left(\frac{1}{D_4}-\frac{1}{D_2}\right)f=\frac{1}{a_3a_4}\left(\frac{1}{D_3D_4}-\frac{1}{D_3D_2}-\frac{1}{D_1D_4}+\frac{1}{D_1D_2}\right)f=$$
$$=\frac{1}{a_3a_4}\left(\frac{1}{(D_3-D_4)}\left(\frac{1}{D_4}-\frac{1}{D_3}\right)-\frac{1}{(D_3-D_2)}\left(\frac{1}{D_2}-\frac{1}{D_3}\right)-\right.$$
$$\left.-\frac{1}{(D_1-D_4)}\left(\frac{1}{D_4}-\frac{1}{D_1}\right)+\frac{1}{D_1D_2}\right)f=$$
$$=\frac{1}{a_3a_4}\left(\frac{1}{a_4-a_3}\left(\frac{1}{D_4}-\frac{1}{D_3}\right)-\frac{1}{(-a_3)}\left(\frac{1}{D_2}-\frac{1}{D_3}\right)-\frac{1}{a_4}\left(\frac{1}{D_4}-\frac{1}{D_1}\right)+\frac{1}{D_1D_2}\right)f=$$
$$=\frac{1}{a_3a_4}\left(\frac{1}{a_4}\frac{1}{D_1}+\frac{1}{a_3}\frac{1}{D_2}-\frac{a_4}{(a_4-a_3)a_3D_3}+\frac{a_3}{(a_4-a_3)a_4D_4}D_4+\frac{1}{D_1D_2}\right)f\,.$$

Partial solutions $F_{1r}(x,y)$ and $F_{2r}(x,y)$ of first two equations (21) can differ by the arbitrary harmonic function, but since our concern is with the arbitrary partial solution of equation (19), we can assume that $F_{1r}(x,y)=F_{2r}(x,y)$. Then taking (21) into consideration, we obtain the final expression for the partial solution of equation (19):

$$F_r(x,y)=\frac{1}{a_3a_4}\left(\frac{(a_3+a_4)}{a_4a_3}F_{1r}-\frac{a_4}{(a_4-a_3)a_3}F_{3r}+\frac{a_3}{(a_4-a_3)a_4}F_{4r}+F_{5r}\right).\quad(22)$$

Thus, partial solution of partial-derivative differential equation of 8th order (19) is represented by the linear combination of partial solutions of as follows: Poisson's equation, two heterogeneous Helmholtz equations and heterogeneous bi-harmonic equation.

The general solution of the system (10) (and (12)) with consideration of (14), (16) – (18), (22) is expressed through the general solutions of bi-harmonic equation and two Helmholtz equations, and partial solutions of heterogeneous bi-harmonic equation, Poisson's equation and two heterogeneous Helmholtz equations.

The general solution of the system (9) (and (5)) is expressed through the general solutions of two Helmholtz equations, and general solution of the system (11) is expressed through the general solutions of two Helmholtz equations and partial solutions of two heterogeneous Helmholtz equations.

The integration constants included in the general solution of the system (4) at every approximation in terms of the small parameter, are defined by the lateral surface boundary conditions.

Remark. The used operator method here is implied also by method of differential equations [7, P. 60-66; 8, P. 154-159] order reduction.

6. On convergence of series (1) and (8). Let us formulate (without proof) the series (1) and (8) convergence theorem.

Let us denote the closed domain of three variables x, y, z, which is occupied by shell, as $\overline{C}_V$ (x, y – tangential coordinates, z – transversal coordinate: $-h/2 \le z \le h/2$), and respective x, y variation domain as $\overline{C}_D$.

Theorem 1 (about convergence of series (1)). If the functional series $\sum_{k=0}^{\infty} u_k(x, y)$ is uniformly and absolutely convergent in the $\overline{C}_V$ domain, then series $\sum_{k=0}^{\infty} P_k(2z/h) u_k(x, y)$ is also uniformly and absolutely convergent in the $\overline{C}_V$ domain.

Theorem 2 (about convergence of series (8)). If functions $u_{0i}(x, y)$ are uniformly bounded in the $\overline{C}_D$ domain, then series $\sum_{i=0}^{\infty} \varepsilon^i u_{0i}(x, y)$ is uniformly and absolutely convergent in this domain.

The formulated theorems are true for other series (1) and (8).

7. Conclusions. Applied equation rearrangement method, use of operator method and method of perturbations of geometrical parameters result in reduction of resolving heterogeneous partial-derivative differential equation system of 16th order within the variant of mathematical non-thin transversely isotropic shells theory to the solution of equations of the second and fourth orders (Laplacian and Poisson's equations, homogeneous and heterogeneous Helmholtz equations, bi-harmonic and heterogeneous bi-harmonic equations). The offered method enables considerable simplification of solution of boundary problems for the non-thin shallow shells and could be also extended to solution of problems for shells in the framework of other theories.

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